CHAPTER SIX

POLYGON

Polygons:

- A polygon is a plane figure bounded by straight lines.

Polygon	
Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

The interior and exterior angles of a polygon:

- The interior angles of a polygon are those angles, which lie within the polygon. -The exterior angles of a polygon are those ones, which lie outside the polygon.

е Example:

I = interior angle.

E = exterior angle.

- For any polygon, the sum of the exterior angles = 360° .

Q1) Calculate the value of each exterior angle of a regular decagon.

Soln

Decagon has 10 sides and as such has 10 exterior angles. But since the sum of the exterior angles of any polygon = 360° ,

 \Rightarrow 10 exterior angles = 360°,

 \Rightarrow 1 exterior angle = $\frac{1}{10} \times 360 = 36^{\circ}$.

 \Rightarrow Each exterior angle = 36°.

Q2) Find the value of each exterior angle of a regular pentagon.

Soln

Since pentagon has 5 sides, then it has 5 exterior angles.

But since the sum of the exterior angles of a polygon = 360° ,

$$\Rightarrow$$
5 exterior angles = 360°,

 \implies 1 exterior angle = $\frac{1}{5} \times 360^\circ = 72$.

The value of each exterior angle of a pentagon = 72°

Q3).



For the given figure, determine the value of *x*.

Soln

The given figure is a quadrilateral or a polygon. The angles marked $5x^{\circ}, 4x^{\circ}, 3x^{\circ}$ and $8x^{\circ}$ are the exterior angles, and since the sum of the exterior angles of a polygon = $360^{\circ}, \Rightarrow 3x + 8x + 5x + 4x = 360^{\circ}, \Rightarrow 20x = 360 \Rightarrow x = \frac{360}{20} = 18^{\circ}.$

Q4.



For the given figure, determine

- a) the value of x.
- b) the values of the angles marked x, 2x, 3x, 4x and 5x.

Soln

(a)The given figure has five sides and as such it is a pentagon which is a polygon. The angles marked x° , $2x^{\circ}$, $3x^{\circ}$, $4x^{\circ}$, and $5x^{\circ}$ are the exterior angles. Since the sum of the exterior angles of a polygon = 360° , then $x^\circ + 2x^\circ + 3x^\circ + 4x^\circ + 5x^\circ = 360^\circ$, $\Rightarrow 15x^\circ = 360^\circ$, $\Rightarrow x = \frac{360}{15} = 24^\circ$.

a) i. The angle marked x° = 24°.
ii. The angle marked 2x = 2(24) = 48°.

iii. The angle marked $3x = 3(24) = 72^{\circ}$.

- i. The angle marked $4x = 4(24) = 96^{\circ}$.
- ii. The angle marked $5x = 5(24) = 120^{\circ}$.

(Q5)



For the given figure, determine the values of the angles marked x and 3x.

Soln.

The given figure is a polygon and the angles marked x° , x° , $3x^{\circ}$, 30° and 80° are the exterior angles. Since the sum of the exterior angles = 360° , => $x^{\circ} + x^{\circ} + 3x^{\circ} + 30^{\circ} + 80^{\circ} = 360^{\circ}$, => $5x + 110^{\circ} = 360$, => 5x = 360 - 110 = 250, = > $x = \frac{250}{5} = 50 => x = 50^{\circ}$.

The value of the angle marked $x = 50^{\circ}$, and that of the angle marked $3x = 3(50) = 150^{\circ}$

Q6.



For this figure, calculate

- a) the value of *x*.
- b) the values of the angles marked *x*, 2x and $3x^{\circ}$.

N/B: The value of the exterior angle which lies next to the 130° angle must first be determined. Let this angle = b°

i.e



a) Since angle 130° and b° lie on a straight line, then $130^\circ + b = 180^\circ$, $= b = 180 - 130^\circ = 50^\circ$.

The exterior angles of the given polygon are 40°, $2x^\circ$, b°, 3x and x. Since the sum of the exterior angles of a polygon = 360, => 40 + 2x + b + 3x + x = 360, => 40 + b + 6x = 360, => 40 + 50 + 6x = 360, => 90 + 6x = 360, => 6x = 360 - 90, => $6x = 270^\circ => x = \frac{270}{6} = 45^\circ$.

- a) i. The angle marked x = 45°.
 ii. The angle marked 2x = 2(45°) = 90°.
- iii The angle marked $3x = 3(45^\circ) = 135^\circ$.