CHAPTER FOUR

TRANSFORMATION

- This involves the changing of the position, shape or the size of a figure.

- There are various types and those we shall consider are:

a. Translation. b. Reflection.

c. Rotation. d. Enlargement.

- Reflection, rotation and translation are called rigid motion because under these transformations, the shape or size of the figure transformed does not change
- But enlargement is not a rigid motion since the size of the object changes

TRANSLATION:

- In this, every point moves the same distance in the same direction.
- The sizes of angles, as well as the lengths of lines do not change.
- If the point (x, y) is translated by the vector $\binom{a}{b}, then\ (x, y) -> (x + a, y + b)$;

- Point $(x + a, y + b),$ is the image of the point (x, y).

- The vector $\binom{a}{b}$ is called the translation vector.

- For example if $(2,5)$ is translated by the vector $\binom{1}{4}, then\ (2,5) -> (2 + 1,5 + 4), => (2,5) -> (3,9)$, where (3,9) is the image of (2, 5).

- The transformation mapping can also be presented as: $\binom{x}{y} => \binom{x}{y} + \binom{a}{b} = \binom{x+a}{y+b}, where\ \binom{x}{y} = the$ point which under went the transformation, $\binom{a}{b} = the$ vector of transformation, and $\binom{x+a}{y+b} = the\ image$.

Q1. A point (3, 1) underwent a translation. If the translation vector is $\binom{4}{2}$, determine its image.

Soln.

Under a translation by the vector $\binom{4}{2}, (3,1) -> (3+4, 2+1), => (3,1) -> (7,3)$.

Q2. Determine the image of the point (-2, 4), under a translation by the vector $\binom{-1}{-3}$

Soln.

Under such a translation, (-2, 4) $-> (-2 +_{1}^{-}, 4 +_{3}^{-}) => (-2,4) -> (-2-1, 4-3). => (-2,4) -> (-3,1)$.

Q3. If the point $\binom{-3}{-2}$ undergoes a translation by the vector $\binom{2}{-4}$, determine its image.

Soln.

Under such a transformation, $\binom{-3}{-2} -> \begin{pmatrix} -3+2 \\ -2+_{4}^{-} \end{pmatrix}, => \binom{-3}{-2} -> \binom{-1}{-6}$.

Q4. If p* (4, 6) is the image of a point p, under a translation by vector $\binom{1}{2}, determine$ the coordinates of the point p.

Soln.

If the coordinates of p is $\begin{pmatrix} x \\ y \end{pmatrix}$, then under such a translation, $\begin{pmatrix} x \\ y \end{pmatrix} ->$ $\begin{pmatrix} x+1 \\ y+2 \end{pmatrix} = \binom{4}{6}$. From $\begin{pmatrix} x+1 \\ y+2 \end{pmatrix} = \binom{4}{6}$, $=> \begin{pmatrix} x \\ y \end{pmatrix} = \binom{4-1}{6-2} = \binom{3}{4}$.

Therefore the coordinates of p are $\binom{3}{4} or\ (3,4)$.

Q5. If Q, (4, 3) is the image of the point Q under a translation by the vector $\binom{-2}{-4}$, find the coordinates of Q.

Soln.

Let $\begin{pmatrix} x \\ y \end{pmatrix}$ = the coordinates of the point Q.

Under such a translation, $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x+\bar{2} \\ y+\bar{4} \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \therefore \begin{pmatrix} x-2 \\ y-4 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ => $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4+2 \\ 3+4 \end{pmatrix}$, => $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ => the point Q has coordinates (6, 7).

Q6. As a result of a translation, the image of $A_1\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ *was* $A_{11}\begin{pmatrix} 5 \\ 11 \end{pmatrix}$; Find the vector of translation.

Soln.

Let $\begin{pmatrix} x \\ y \end{pmatrix}$ = the vector of translation. *Then* $\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$, => $\begin{pmatrix} 2+x \\ 4+y \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$, => $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5-2 \\ 11-4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$, =>the vector of translation = $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$.

Q7. After being subjected to a translation, the image of the point $p\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ was $p_1\begin{pmatrix} -6 \\ -2 \end{pmatrix}$. What was the translation vectors?

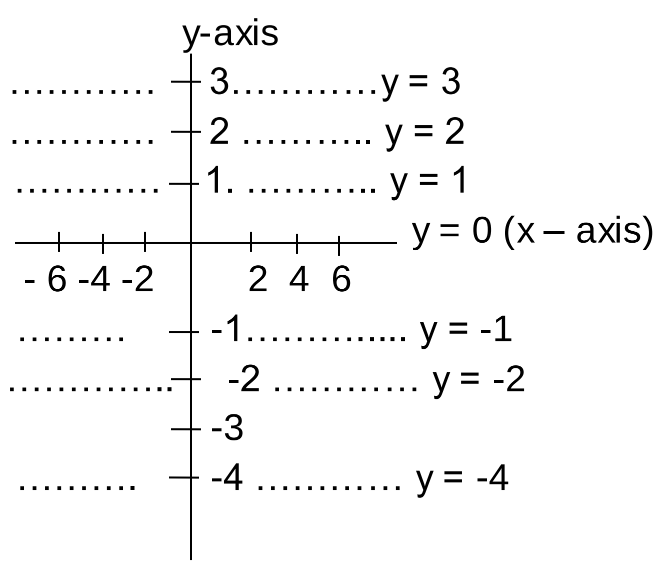
Soln.

Let $\begin{pmatrix} x \\ y \end{pmatrix}$ = the translation vector. Then $\begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$, => $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6+2 \\ -2-4 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$ =>the translation vector is $\begin{pmatrix} -4 \\ -6 \end{pmatrix}$.

Reflection:

- Reflection occurs literally in the mirror line, which is the position where the mirror is assumed to be positioned.
- With respect to reflection, the object and its image are on the opposite side of the mirror line.
- Apart from that, the perpendicular distance between the object and this line, is the same as the perpendicular distance between the line and the image.

N/B:



- The - x axis is the same as the line y = 0.
- Under reflection, the lengths of lines and the sizes of angles do not change.

Reflection in the x-axis or the line y = 0:

- For such a reflection,$(x, y) -> (x, -y)$.
- This mapping can also be written as $\binom{x}{y} -> \binom{x}{-y}$

Q1. If the point (2, 4) undergoes a reflection in the line y = 0, determine its image.

Soln.

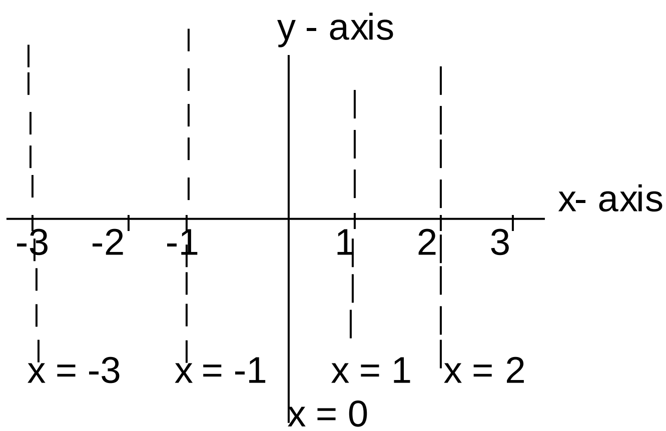
Under such a reflection(x, y) → (x,- y), => (2, 4) → (2, - 4).

Q2. Determine the image of the point (-2,- 4), after a reflection in the x – axis.

Soln.

Under such a reflection, (x, y) →(x, -y),=>(-2,-4) → (-2, 4).

N/B:



The y- axis is the same as the line x = 0.

Reflection in the y-axis or the line x = 0:

- Under such a reflection, (x, y) →(-x, y).
- This mapping can also be written as $\binom{x}{y} -> \binom{-x}{y}$.

Q1. Determine the image of the point (2, 4), under a reflection in the y – axis.

Soln.

Under such a reflection, (x, y) $->$ (-x, y),=> (2, 4) $->$ (-2, 4).

Q2. Find the image of the point $\binom{-2}{-6}$, under a reflection in the line x = 0.

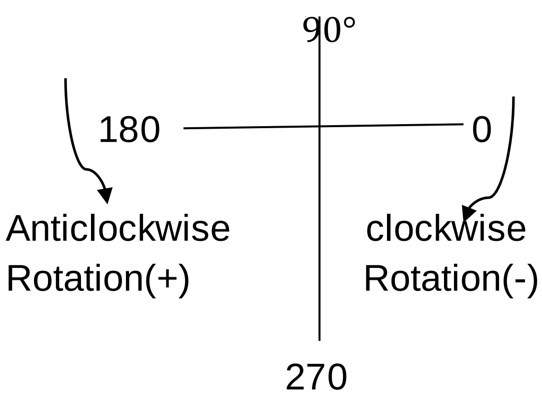
Soln.

For such a reflection, (x, y) $\rightarrow$ (-x, y),=>(-2,-6) $\rightarrow$ (2,-6).

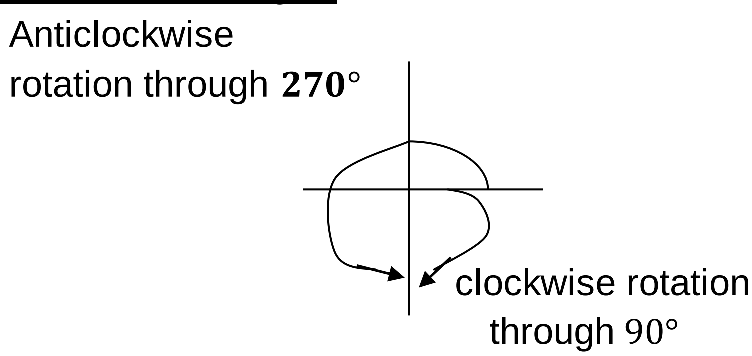
Rotation:

- This is measured in degrees, and in either the anticlockwise or the clockwise direction.
- Rotation in the clockwise direction is taken as negative, while that in the anticlockwise direction is taken as positive.

i.e



- Rotation is measured from the x-axis, or with reference to the x – axis.
- **Clockwise rotation through $90°$ or anticlockwise rotation through $270°$about the origin:**



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- From the drawn diagram, a clockwise rotation through $90°$, is the same as an anticlockwise rotation through $270°$,since they all have a common meeting point or meet on the same line.
- Under such a rotation (x, y) $\longrightarrow$ (y, -x).