CHAPTER SIX

VECTORS

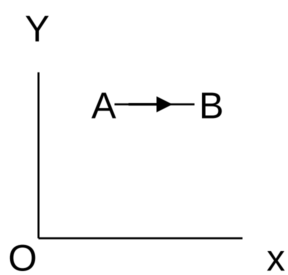
- A vector is any physical quantity which has both magnitude and direction.
- Examples of vectors are:
  a. a force of 20N acting north.
  b. a velocity of 5km/h East.

Types of vectors:

- In general there are two types of vectors, and these are:

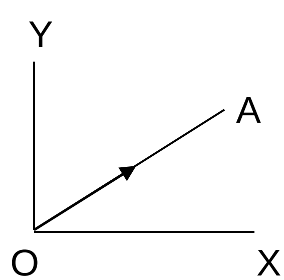
i. Free vector.
ii. Position vector.

Free vector:



- A free vector is a vector which does not pass through any specific point or position.
- They are usually represented by small letters e.g. $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$.

Position vector:



This is a vector which passes through the origin or a specified point.

Vector Notation:

- A vector can be represented by a line segment as shown next:

A $\longrightarrow$ B

- This may be written as $\overrightarrow{AB}$, $\overline{AB}$, $\underline{AB}$, $\widetilde{AB}$ or $\underset{\sim}{AB}$.

The unit vector ;

- This is a vector whose magnitude is one in a direction under consideration.

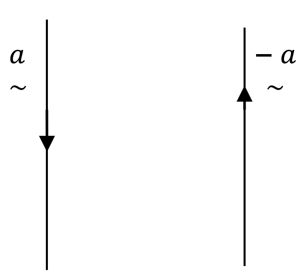
- The unit vector along a vector $\vec{a}$ is written as â.
- Also the unit vector along the vector $\overline{AB}$ is written as $\widehat{AB}$.

The zero vector or the null vector:

- This is a vector whose magnitude is zero and its direction is undefined.
- It is represented by $\underset{O}{\rightarrow} = \binom{o}{o}$
- A zero vector has no effect with respect to addition.
- Therefore if $\underset{\sim}{O}$ = the zero vector, then $\underset{\sim}{a} + \underset{\sim}{O} = \underset{\sim}{a}$

The negative vector:

- The negative vector of the vector $\underset{\sim}{a}$ is written as - $\underset{\sim}{a}$
- A vector when added to its negative gives us the zero vectors.
- For example $\underset{\sim}{a} + (-\underset{\sim}{a}) = \underset{\sim}{O}$
- The vector $-\underset{\sim}{a}$ is a vector which has the same magnitude as $\underset{\sim}{a}$, but it is in the opposite direction.



- If $\underset{\sim}{a} = \overline{AB}$ $then$ $-\underset{\sim}{a} = \overline{BA}, and\ \overrightarrow{AB} + \overrightarrow{BA} = \underset{\sim}{O}$
- Also if $\underset{\sim}{b} = \overrightarrow{CD},\ then\ -\underset{\sim}{b} = \overrightarrow{DC}\ and\ \overrightarrow{CD} + \overrightarrow{DC} = \underset{\sim}{O}$

N/B: The negative vector of the vector $\overrightarrow{CD}\ is\ \overrightarrow{DC}$.

Notation of the magnitude of a vector:

- The magnitude of the vector $\overrightarrow{AB}$ is written as
- $|\overrightarrow{AB}|$
- Similarly the magnitude of the vector $\vec{b}$ is written as| b |,
- If vector $\overrightarrow{OP} = \binom{a}{b},\ then\ its\ magnitude = |\overrightarrow{OP}| = \sqrt{a^2 + b^2}$
- For example if $\overrightarrow{OP}$ = $\binom{6}{5}, then\ the\ magnitude\ of\ \overrightarrow{OP} = \sqrt{6^2 + 5^2} = \sqrt{61}$

Scalar multiplication of a vector:

- A scalar may be a whole number or a fraction.
- When a scalar multiplies a vector, the product or what we get is also a vector.
- For example if our vector is $\tilde{a}$ and the scalar is 2, then 2 x $\tilde{a}$ = 2$\tilde{a}$ and 2$\tilde{a}$ is also a vector.

Q1. Find the numbers m and n such that $m\binom{3}{5} + n\binom{2}{1} = \binom{4}{9}$.

Soln.

$$M\begin{pmatrix}3\\5\end{pmatrix} + n\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}4\\9\end{pmatrix} => \begin{pmatrix}3m\\5m\end{pmatrix} + \begin{pmatrix}2n\\n\end{pmatrix} = \begin{matrix}4\\9\end{matrix}\ , therefore\ \ 3m + 2n = 4\ and\ 5m + n = 9$$

Solve these two equations simultaneously i.e

Let 3m + 2n = 4 ----- eqn (1)

and 5m + n = 9 ------ eqn (2)

Equation (2) x -2 => $-10m - 2n = -18 \ldots\ldots.. eqn(3)$

Add eqn (1) and eqn (3)

i.e $3m + 2n = 4$

$$\begin{array}{r} 3m + 2n = 4 \\ + \underline{-10m - 2n = -18} \\ \underline{-7m + 0 = -14} \end{array}$$

$=> -7m = -14, => m = \frac{-14}{-7}, => m = 2.$

Put m = 2 into equation (2) ie 5m + n = 9 $=> 5(2) + n = 9, => 10 + n = 9, => n = 9 - 10 = -1.$

N/B:

i. Given the points A and B, then $\overrightarrow{AB} = B - A$.
ii. For example if A = $\binom{5}{2}$ $and\ B = \binom{10}{6}, then\ \overrightarrow{AB} = B - A = \binom{10}{6} - \binom{5}{2} = \binom{10-5}{6-2} = \binom{5}{4}$.
iii. Also if C = $\binom{4}{2}$ $and\ D = \binom{6}{1}, then\ \overrightarrow{CD} = D - C = \binom{6}{1} - \binom{4}{2} = \binom{6-4}{1-2} = \binom{2}{-1}$

Q2. If A = (4, 5) and B = (6, 2) find $\overrightarrow{AB}$

Soln.

Since A = (4, 5) => A = $\binom{4}{5}$.

Also since B = (6, 2) $=> B = \binom{6}{2}. \overrightarrow{AB} = B - A = \binom{6}{2} - \binom{4}{5} = \binom{6-4}{2-5} = \binom{2}{-3}, => \overrightarrow{AB} = \binom{2}{-3}$.

N/B:

i. $\overrightarrow{AB} = -\overrightarrow{BA}$
ii. If $\overrightarrow{AB} = \binom{4}{2}, then\ \overrightarrow{BA} = -AB = -\binom{4}{2} = \binom{-4}{-2}$
iii. Also if $\overline{CD} = \binom{-2}{5}, then\ \overline{DC} = -\overline{DC} = -\binom{2}{-5}$

Q3. If A and B are the points (2,1) and (1, 2) respectively, find $\overline{AB}\ and\ \overline{BA}$.

Soln.

i. Since A = $\binom{2}{1}$ $and\ B = \binom{1}{2}, => \overline{AB} = B - A = \binom{1}{2} - \binom{2}{1} = \binom{1-2}{2-1} = \binom{-1}{1}, => \overline{AB} = \binom{-1}{1}$..

i. $\overline{BA} = -\overline{AB} = -\binom{-1}{1} = \binom{1}{-1}$.

Q4. If $\vec{A} = \binom{2}{1} and\ \vec{B} = \binom{3}{4}$, evaluate the following:

i. $\vec{A} + \vec{B}$ ii. $\left|\vec{A} + \vec{B}\right|$

iii. $\overrightarrow{AB}$ iv. $\left|\overrightarrow{AB}\right|$

Soln.

i. $\vec{A} + \vec{B} = \binom{2}{1} + \binom{3}{4} = \binom{2+3}{1+4} = \binom{5}{5}$.

ii. $Since\ \vec{A} + \vec{B} = \binom{5}{5}, => \left|\vec{A} + \vec{B}\right| = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50}$.

iii. $\overline{AB} = B - A = \binom{3}{4} - \binom{2}{1} = \binom{3-2}{4-1} = \binom{1}{3}$

iv. $Since\ \overline{AB} = \binom{1}{3}, => \left|\overline{AB}\right| = \sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$.

N/B: $\left|\overline{AB}\right|$ means the magnitude of vector AB.